ASSIGNMENT-1

M.Sc.(Maths) 2nd Semester

Subject-Measure Theory and Integration

Prove that intersection and difference of two measurable sets is measurable.

1. Show that if E_1 and E_2 are measurable sets, then

   $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$.

2. Let M be the collection of all measurable sets of R and $< E_n >$ is any sequence of sets in

   M then $m(\cup E_n) \leq \Sigma m(E_n)$.

3. If A and B are any two disjoint subsets of R then prove that

   $m^*(A \cup B) = m^*(A) + m^*(B)$.

4. If E_1, E_2 and E_3 are measurable sets then prove that

   $m(E_1) + m(E_2) + m(E_3) + m(E_1 \cap E_2 \cap E_3) = m(E_1 \cup E_2 \cup E_3) + m(E_1 \cap E_2) + m(E_2 \cap E_3)$
   $+ m(E_1 \cap E_3)$

5. If f is a measurable function and $f = g$ almost everywhere, then g is measurable.

6. Determine whether the function defined below is measurable

$$f(x) = \begin{cases} x + 5 & if \quad x < -1 \\ 2 & if \quad -1 \leq x < 0 \\ x^2 & if \quad 0 \leq x \end{cases}$$